

Solution of the η - ^4He problem with quasi-particle formalism

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(Dated: March 21, 2017)

The Alt-Grassberger-Sandhas equations for the five-body $\eta - 4N$ problem are solved for the case of the driving ηN and NN potentials limited to s -waves. The quasi-particle (Schmidt) method is employed to convert the equations into the effective two-body form. Numerical results are presented for the $\eta^4\text{He}$ scattering length.

PACS numbers: 13.75.-n, 21.45.+v, 25.80.-e

I. INTRODUCTION

During the last years considerable attention has been paid to interaction of η -mesons with four nucleons [1–9]. Analysis of different data is mainly focused on the search for $\eta^4\text{He}$ bound states. According to the available experimental results, the rise of the $dd \rightarrow \eta^4\text{He}$ experimental cross section at $E_\eta \rightarrow 0$ seems to be not as steep as in the $pd \rightarrow \eta^3\text{He}$ reaction. As discussed, e.g., in Ref. [3] the most natural interpretation of this fact is that due to additional attraction caused by one extra nucleon the pole in the $\eta^4\text{He}$ scattering matrix is shifted into the region of negative values of $\text{Re } E_\eta$ and turns out to be farther from the physical region than the $\eta^3\text{He}$ pole. It is therefore concluded that formation of the bound $\eta^4\text{He}$ state is highly probable.

In view of general complexity of the five-body $\eta - 4N$ problem there are still no rigorous few-body calculations of this system. At the same time, a systematic practical way of handling the n -body interaction is provided by the quasi-particle formalism in which the kernels of integral equations are represented by series of separable terms. This method becomes especially efficient if the driving two-particle potentials are governed by the nearly lying resonances or bound (virtual) states, like in the NN and ηN case. Then reasonable accuracy may be achieved with only few separable terms retained in the series. In particular, the quasi-particle formalism is shown to be very well suited for practical calculation of ηNN [10–12] as well as $\eta - 3N$ [13] scattering (in Ref. [14] another method based on the hypospherical function expansion has been developed).

In this letter we apply the quasi-particle method to study the five-body system $\eta - 4N$. As a formal basis we use the Alt-Grassberger-Sandhas n -body equations derived in Ref. [15]. For the sake of simplicity we neglect influence of the spin and isospin on the interaction between nucleons, treating them as spinless indistinguishable particles. Furthermore, since only the threshold $\eta^4\text{He}$ energies are considered, we restrict all interactions to s -waves only.

II. FORMALISM

As is well known, separable expansion of the kernels allows one to reduce the n -body integral equations to the $(n - 1)$ -body equations, where two of n particles in each state are effectively treated as a composite particle (quasi-particle). Therefore, the essence of the method is to approximate the $(n - 1)$ -particle interaction obtained in the separable-potential model again by the separable ansatz. In this respect, to simplify presentation of the formalism, we start directly from successive application of the quasi-particle technique to 2-, 3-, and 4-body subamplitudes, occurring when the five-body system is divided into groups of mutually interacting particles.

In what follows, we use the concept of partitions as introduced, e.g., in Ref. [16]. Different partitions (as well as the quasi-particles related to these partitions) are further denoted by the symbols α, β, \dots , whereas the Latin letters a, b, \dots are used for numbering the terms in the separable expansions of the subamplitudes. The notation α_n refers to the partition obtained by dividing the $\eta - 4N$ system into n groups. Writing $\alpha_{n+1} \subset \alpha_n$ means that the partition α_{n+1} is obtained from α_n via further division of the quasi-particle α_n into two groups of particles.

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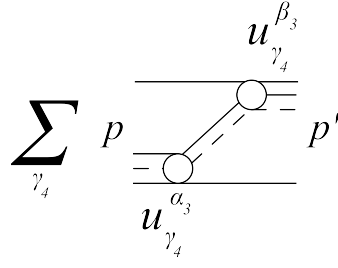


FIG. 1: The potential $Z_{\alpha_3, \beta_3}^{\alpha_2}(z; p, p')$ as defined in Eq. (4) connecting two configurations of the type $(\eta NN) + N$. The dashed and the solid lines represent, respectively, η -mesons and nucleons. The form factors $u_{\gamma_4}^{\alpha_3}$, $u_{\gamma_4}^{\beta_3}$ are shown by the circles.

The basic ingredient of the formalism is a separable expansion of the quasi-particle amplitudes

$$X_{\alpha_{n+1}a, \beta_{n+1}b}^{\alpha_n}(z) = \sum_{k, l=1}^{N_{\alpha_n}} |u_{\alpha_{n+1}(a)}^{\alpha_n(k)}\rangle \Delta_{kl}^{\alpha_n}(z) \langle u_{\beta_{n+1}(b)}^{\alpha_n(l)}|, \quad (1)$$

$$\alpha_{n+1}, \beta_{n+1} \subset \alpha_n.$$

Then the integral equations for the amplitudes $X_{\alpha_n, \beta_n}^{\alpha_{n-1}}$ are transformed exactly into the quasi-two-body equations which in the operator form read

$$X_{\alpha_n a, \beta_n b}^{\alpha_{n-1}} = Z_{\alpha_n a, \beta_n b}^{\alpha_{n-1}} + \sum_{\gamma_n} \sum_{k, l=1}^{N_{\gamma_n}} Z_{\alpha_n a, \gamma_n k}^{\alpha_{n-1}} \Delta_{kl}^{\gamma_n} X_{\gamma_n l, \beta_n b}^{\alpha_{n-1}},$$

$$\alpha_n, \beta_n, \gamma_n \subset \alpha_{n-1}, \quad (2)$$

or more explicitly

$$X_{\alpha_n a, \beta_n b}^{\alpha_{n-1}}(z; p, p') = Z_{\alpha_n a, \beta_n b}^{\alpha_{n-1}}(z; p, p') + \sum_{\gamma_n \subset \alpha_{n-1}} \sum_{k, l=1}^{N_{\gamma_n}} \int \frac{p''^2 dp''}{2\pi^2} Z_{\alpha_n a, \gamma_n k}^{\alpha_{n-1}}(z; p, p'')$$

$$\times \Delta_{kl}^{\gamma_n} \left(z - \frac{p''^2}{2\mu_{\gamma_n}} \right) X_{\gamma_n l, \beta_n b}^{\alpha_{n-1}}(z; p'', p') \quad (3)$$

with μ_{γ_n} being the reduced mass associated with the partition γ_n . The effective potentials $Z_{\alpha_n, \beta_n}^{\alpha_{n-1}}$ are determined as matrix elements of the 'resolvent' $\Delta^{\gamma_{n+1}}$ between the form factors appearing in the expansion (1)

$$Z_{\alpha_n a, \beta_n b}^{\alpha_{n-1}} = \sum_{\gamma_{n+1}} \sum_{k, l} \langle u_{\gamma_{n+1}(k)}^{\alpha_n(a)} | \Delta_{kl}^{\gamma_{n+1}} | u_{\gamma_{n+1}(l)}^{\beta_n(b)} \rangle, \quad (4)$$

$$\gamma_{n+1} \subset \alpha_n, \gamma_{n+1} \subset \beta_n, \quad \alpha_n \neq \beta_n.$$

The structure of Eq. (4) is conveniently illustrated in the form of diagrams. In Fig. 1 we show as an example one of the effective potentials $Z_{\alpha_3, \beta_3}^{\alpha_2}$, connecting two configurations of the type $(\eta NN) + N$. Since the nucleons are identical, the condition $\alpha_n \neq \beta_n$ in Eq. (4) means that the nucleon lines, entering the quasi-particles α_n and β_n and not included into the quasi-particle γ_{n+1} should be different. To calculate the form factors $u_{\gamma_{n+1}(k)}^{\alpha_n(a)}$ and the propagators $\Delta_{kl}^{\gamma_{n+1}}$ we employed the energy dependent pole expansion (EDPE) method of Ref. [17].

A. Four-body partitions

Considering nucleons as indistinguishable particles we have only two different types of four-particle partitions:

$$1: (NN) + N + N + \eta, \quad 2: (\eta N) + N + N + N. \quad (5)$$

The partitions 1 and 2 and the related two-particle subsystems NN and ηN will further be labeled by the index $\alpha_4 = 1, 2$.

TABLE I: The NN potential parameters. E_{NN} , E_{3N} , and E_{4N} are the two-, three-, and four-nucleon binding energies calculated with our model.

Type	λ_{NN} fm ²	β fm ⁻¹	E_{NN} MeV	r_0 fm	E_{3N} MeV	E_{4N} MeV
Yamaguchi	4.17	1.45	0.428	1.89	12.6	54.8
Gauss	6.51	1.24	0.428	2.33	8.05	30.3

In the present calculation, the NN and ηN s -wave interactions were approximated by simplest rank-one separable potentials. For NN we employed

$$v_1(z) = -|g_1\rangle\langle g_1|. \quad (6)$$

The corresponding t -matrix has the usual form

$$t_1(z) = |g_1\rangle\tau_1(z)\langle g_1| \quad (7)$$

with the NN propagator

$$\tau_1(z) = -\left[1 + \frac{1}{2\pi^2} \int_0^\infty \frac{g_1(q)^2}{z - q^2/M_N} q^2 dq\right]^{-1}, \quad (8)$$

where M_N is the nucleon mass. The form factors were chosen in the Yamaguchi form

$$g_1(q) = \frac{\sqrt{\lambda_{NN}}}{1 + (q/\beta)^2}. \quad (9)$$

Since we treat nucleons as spinless particles, the strength λ_{NN} was taken as an average of the singlet and the triplet strength

$$\lambda_{NN} = \frac{1}{2}(\lambda_{NN}^{(0)} + \lambda_{NN}^{(1)}), \quad \lambda_{NN}^{(s)} = \frac{8\pi a_s}{M_N(a_s\beta - 2)}. \quad (10)$$

The singlet and the triplet scattering lengths, a_0 and a_1 , as well as the cut-off momentum β were taken directly from the analysis [18] of the low-energy np scattering

$$a_0 = 23.690 \text{ fm}, \quad a_1 = -5.378 \text{ fm}, \quad \beta = 1.4488 \text{ fm}^{-1}. \quad (11)$$

It is well known that the Yamaguchi NN potential overestimates attraction at high momenta and yields significant overbinding already in the ^3He case (see Table I). Therefore we also adopted the spin-independent NN potential with exponential form factors

$$g_1(q) = \sqrt{\lambda_{NN}} e^{-q^2/\beta^2}, \quad (12)$$

which yields the same binding energy E_{NN} of two nucleons. The form factors (12) with parameters listed in Table I give for the three- and four-nucleon binding energies, E_{3N} and E_{4N} , the values which are rather close to those of the ^3He and ^4He nuclei. At the same time, with the Gauss form factors we have a visibly larger value of the NN effective range r_0 (see Table I).

The ηN s -wave interaction was reduced to excitation of the resonance $N(1535)1/2^-$ only. To include pions we used a conventional coupled channel formalism, where the resulting separable t -matrix has the matrix form

$$t_{\mu\nu}(z) = \frac{1}{W - M_0} |g_\mu\rangle\tau_2(z)\langle g_\nu|, \quad \mu, \nu \in \{\pi, \eta\} \quad (13)$$

with

$$g_\mu(q) = \frac{g_\mu}{1 + (q/\beta_\mu)^2}. \quad (14)$$

The propagator

$$\tau_2(z) = \frac{1}{W - M_0 - \Sigma_\eta(W) - \Sigma_\pi(W) + \frac{i}{2}\Gamma_{\pi\pi}(W)}$$

TABLE II: The $\eta N - \pi N$ parameters.

Set [Ref.]	g_η	β_η MeV	g_π	β_π MeV	M_0 MeV	$\gamma_{\pi\pi}$ MeV
I [19]	1.91	636	0.651	850	1577	4.0
II [20]	1.23	636	1.28	350	1527	1.0

with $W = z + M_N + M_\eta$, where M_η is the η mass, is determined by the $N(1535)1/2^-$ self-energies $\Sigma_\eta(W)$ and $\Sigma_\pi(W)$. The two-pion channel was included via the $\pi\pi N$ decay width $\Gamma_{\pi\pi}$ parametrized in the form

$$\Gamma_{\pi\pi}(W) = \gamma_{\pi\pi} \frac{W - M_N - 2M_\pi}{M_\pi}. \quad (15)$$

The parameters g_η , β_η , g_π , β_π , M_0 , and $\gamma_{\pi\pi}$ were chosen in such a way that the scattering amplitude $f_{\eta N}$ corresponding to our t -matrix $t_{\eta\eta}$ (13) is close to that obtained in the coupled-channel analyses in the energy region from 20 MeV above the ηN threshold to 100 MeV below the threshold. Here we took the results of two works [19] and [20] predicting rather different values of $Re f_{\eta N}$ (see Fig. 2).

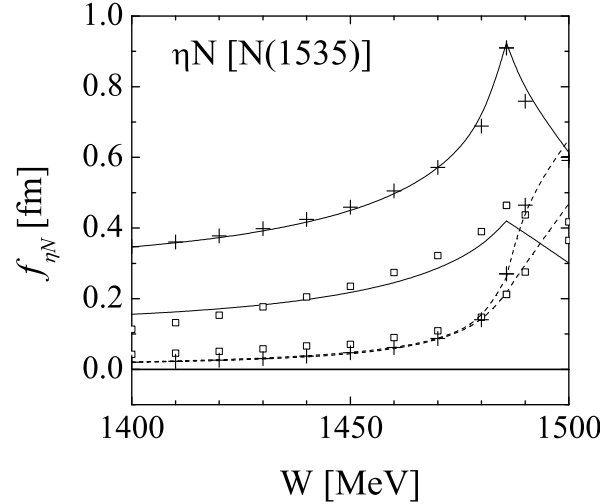


FIG. 2: The S_{11} partial wave of the ηN scattering amplitude calculated with Sets I and II of the parameters listed in Table II. Notations: solid curve: real part, dashed curve: imaginary part. Crosses and squares represent the results of the coupled channel analysis of Refs. [19] and [20], respectively.

B. Three-body partitions

We have four different three-body partitions

$$\begin{aligned} 1 : (NNN) + N + \eta, & \quad 2 : (\eta NN) + N + N, \\ 3 : (\eta N) + (NN) + N, & \quad 4 : (NN) + (NN) + \eta \end{aligned} \quad (16)$$

which in the following are numerated by the index $\alpha_3 = 1, \dots, 4$. In the latter two cases there are two pairs of interacting particles propagating independently. The effective potentials $Z_{\alpha_4, \beta_4}^{\alpha_3}$ determined by Eq. (4) for $n = 4$ are matrix elements of the free resolvent G_0 between the form factors g_{α_4} ($\alpha_4 = 1, 2$)

$$Z_{\alpha_4, \beta_4}^{\alpha_3} = \langle g_{\alpha_4} | G_0 | g_{\beta_4} \rangle. \quad (17)$$

The functions $g_{\alpha_4}(q)$ are given by Eqs. (9) (or (12)) and (14) with $g_2(q) \equiv g_\eta(q)$. Here we omit the superfluous indices a, b , since our separable ansatz for NN and ηN amplitudes contains in both cases only one term (see Eqs. (7) and (13)).

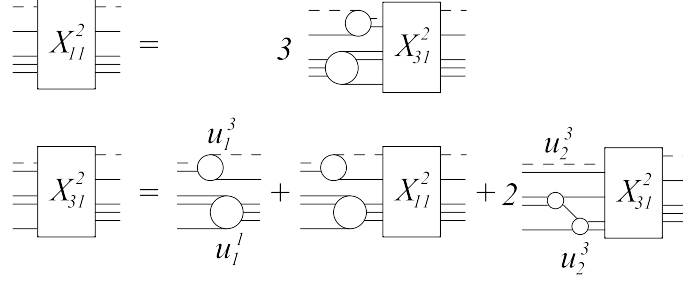


FIG. 3: Effective quasi-two-body equations for the $(\eta N) - (NNN)$ amplitudes $X^2_{\alpha_3, \beta_3}$. Notation of the lines as in Fig. 1. The lower and the upper indices in $u_{\alpha_4}^{\alpha_3}$ refer to the numbers of the four- and three-body partitions listed in Eqs. (5) and (16), respectively. The numerical coefficients appear due to symmetrization of the nucleon states.

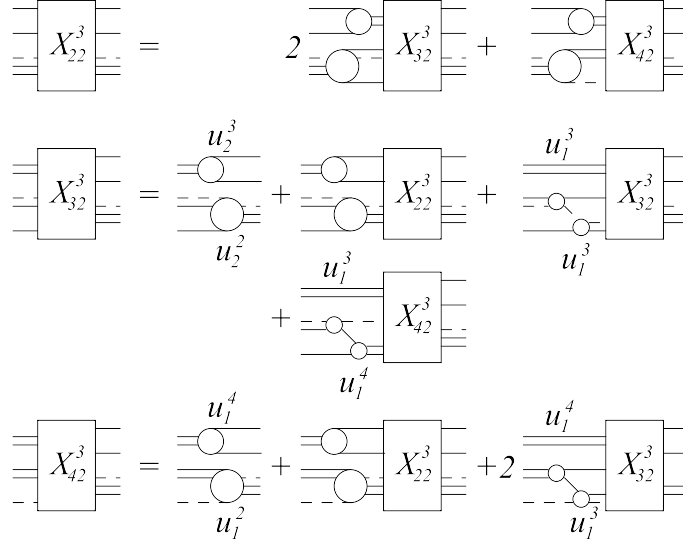


FIG. 4: Same as in Fig. 3 for the $(\eta NN) - (NN)$ amplitudes $X^3_{\alpha_3, \beta_3}$.

C. Two-body partitions

There are four two-body partitions of the $\eta - 4N$ system:

$$\begin{aligned} 1 : & \eta + (NNNN), & 2 : & (\eta N) + (NNN), \\ 3 : & (\eta NN) + (NN), & 4 : & (\eta NNN) + N \end{aligned} \quad (18)$$

which will be labeled by $\alpha_2 = 1, \dots, 4$.

The effective potentials $Z_{\alpha_3 a, \beta_3 b}^{\alpha_2}$ are matrix elements of the 'resolvent' τ_{α_4} between the form factors $u_{\alpha_4}^{\alpha_3(a)}$ appearing in the separable expansion (1) for $n = 3$:

$$Z_{\alpha_3 a, \beta_3 b}^{\alpha_2} = \sum_{\gamma_4=1,2} \langle u_{\gamma_4}^{\alpha_3(a)} | \tau_{\gamma_4} | u_{\gamma_4}^{\beta_3(b)} \rangle. \quad (19)$$

The propagators τ_{α_4} ($\alpha_4 = 1, 2$) are given by (8) and (15) with $\tau_2 \equiv \tau_{\eta\eta}$.

The calculation of the $NNNN$ ($\alpha_2 = 1$) and ηNNN ($\alpha_2 = 4$) amplitudes with separable NN potentials may be found, e.g., in Refs. [21] and [13], and we refer the reader to these works. The effective $(3+2)$ amplitudes ($\alpha_2 = 2, 3$) describe propagation of two groups of mutually interacting particles. The corresponding integral equations are schematically presented in Figs. 3 and 4.

After the separable expansions (1) for $n = 2$ are calculated we build the effective potentials $Z_{\alpha_2 a, \beta_2 b}$ (4) as

$$Z_{\alpha_2 a, \beta_2 b} = \sum_{\gamma_3=1}^4 \sum_{k,l=1}^{N_{\gamma_3}} \langle u_{\gamma_3(k)}^{\alpha_2(a)} | \Delta_{kl}^{\gamma_3} | u_{\gamma_3(l)}^{\beta_2(b)} \rangle. \quad (20)$$

The corresponding system of the five-body $\eta-4N$ equations is diagrammatically presented in Fig. 5. After this system is solved, the $\eta^4\text{He}$ scattering amplitude can be calculated as

$$f_{\eta^4\text{He}}(p) = -N^2 \frac{\mu}{2\pi} X_{11,11}(z; p, p). \quad (21)$$

Here N is the normalization constant of the ^4He wave function, μ is the $\eta-^4\text{He}$ reduced mass, and the momentum p is fixed by the on-mass-shell condition

$$p = \sqrt{2\mu(z + E_{4N})}, \quad (22)$$

where $E_{4N} > 0$ is the four-nucleon binding energy given in Table I.

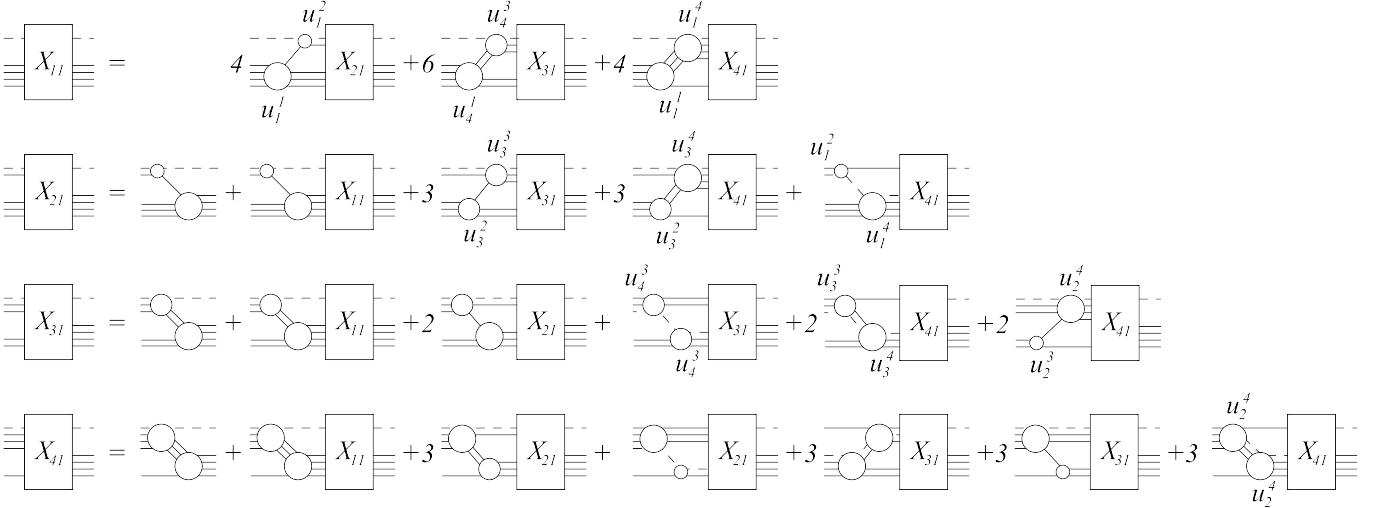


FIG. 5: Graphical representation of the effective quasi-two-body equations for $\eta-4N$ scattering. Notations as in Fig. 1. The lower and the upper indices in $u_{\alpha_3}^{\alpha_2}$ refer to the three- and two-body partitions, as given in Eqs. (16) and (18). The numerical factors arise from the identity of the nucleons.

In Table III we present the value of the $\eta^4\text{He}$ scattering length calculated with different number N_{α_2} of terms retained in the separable expansion (1) of the amplitudes $X_{\alpha_3, \beta_3}^{\alpha_2}$. As one can see, satisfactory accuracy is achieved with $N_1 = N_2 = 6$, $N_3 = N_4 = 8$. In principle, already with first four terms in each expansion the resulting scattering length is within less than 2% of the correct value. Thus, also in the five-body case $\eta-4N$ the quasi-particle approach based on the EDPE method of Ref. [17] is very suitable for practical applications. The minimum number of separable terms N_{α_2} only slightly exceeds that for the four-body kernels, where convergence is achieved already with first four-six terms in each subamplitude.

III. DISCUSSION AND CONCLUSION

As our main result we present the $\eta^4\text{He}$ scattering length $a_{\eta^4\text{He}} = f_{\eta^4\text{He}}(0)$. It is given in Table IV for two versions of the NN potential. For comparison purposes also the $\eta^3\text{He}$ scattering length calculated with the same sets of the NN and $\eta N - \pi N$ parameters is presented.

TABLE III: The scattering length $a_{\eta^4\text{He}}$ as a function of N_{α_2} ($\alpha_2 = 1, \dots, 4$), the number of separable terms retained in the separable expansion (1) for the (4+1) and (3+2) subamplitudes X^{α_2} . The calculation is performed with the Gauss NN potential and Set I of the $\eta N - \pi N$ parameters.

N_1	N_2	N_3	N_4	$a_{\eta^4\text{He}}$ [fm]
2	2	2	2	$5.56 + 0.96 i$
4	4	4	4	$4.88 + 1.23 i$
4	4	6	6	$4.83 + 1.23 i$
6	6	8	8	$4.79 + 1.22 i$
10	10	12	12	$4.79 + 1.22 i$
20	20	20	20	$4.80 + 1.22 i$

It is remarkable, that despite the larger number of nucleons in ^4He the predicted value of $a_{\eta^4\text{He}}$ is smaller than $a_{\eta^3\text{He}}$. Direct calculation shows that the main reason of this somewhat unexpected result is rather rapid decrease of the ηN scattering amplitude in the subthreshold region (see Fig. 2). Because of essentially stronger binding of ^4He in comparison to ^3He , in the former case the effective in-medium ηN interaction acts at lower internal ηN energies, thus leading to general reduction of the attractive ηN forces (this question was addressed in detail in Refs. [22–24]). This effective weakening may qualitatively explain why the peculiar slope in the η spectrum at low energies seen in the data for $dd \rightarrow \eta^4\text{He}$ [7] and $pd \rightarrow \eta^3\text{He}$ [25, 26] becomes less steep, when we turn from $\eta^3\text{He}$ to $\eta^4\text{He}$.

TABLE IV: The $\eta^3\text{He}$ and $\eta^4\text{He}$ scattering lengths predicted by our calculation. The first and the second rows for each version of the NN potential list the values obtained with Set I and Set II of the $\eta N - \pi N$ parameters, respectively.

NN	$\eta N - \pi N$	$a_{\eta^3\text{He}}$ [fm]	$a_{\eta^4\text{He}}$ [fm]
Yamaguchi	I	$6.5 + 3.6 i$	$2.2 + 0.3 i$
	II	$1.1 + 0.5 i$	$0.5 + 0.1 i$
Gauss	I	$6.7 + 4.0 i$	$4.8 + 1.2 i$
	II	$1.3 + 0.7 i$	$1.0 + 0.3 i$

Summarizing, $\eta^4\text{He}$ interaction is calculated for the first time correctly dealing with the few-body aspects of the problem. Applying separable representation firstly to the (3+1) and (2+2) and then to the (4+1) and (3+2) kernels we have solved the five-body Alt-Grassberger-Sandhas equations reducing them to a coupled set of quasi-two-body equations having Lippmann-Schwinger structure.

The predicted value of $Re a_{\eta^4\text{He}}$ is positive and turns out to be smaller than $Re a_{\eta^3\text{He}}$. This finding should be attributed to effective weakening of the in-medium ηN interaction. According to our calculation, increase of the attractive forces due to an extra nucleon in ^4He is overwhelmed by stronger suppression of the subthreshold ηN interaction in a more dense nucleus. The resulting attraction in the $\eta - 4N$ system is too weak and does not support existence of the $\eta^4\text{He}$ bound state, at least with the ηN parameters, used in the present calculation. This might be the key reason why no signal of $\eta^4\text{He}$ bound state formation is still revealed, e.g., in the $dd \rightarrow ^3\text{He} n \pi^0$ and $dd \rightarrow ^3\text{He} p \pi^-$ reactions [27, 28].

Finally, we note that although our results obviously suffer from oversimplified treatment of the NN potential, they demonstrate applicability of the quasi-particle formalism to the five-body $\eta^4\text{He}$ problem. The EDPE method provides rather rapid convergence of the separable expansion, so that transition from $\eta - 3N$ to the $\eta - 4N$ case is performed without drastic increase of numerical complexity. At the same time, more refined treatment requires inclusion of the nucleon spin as well as more sophisticated nucleon-nucleon potential instead of our simple rank-one ansatz.

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